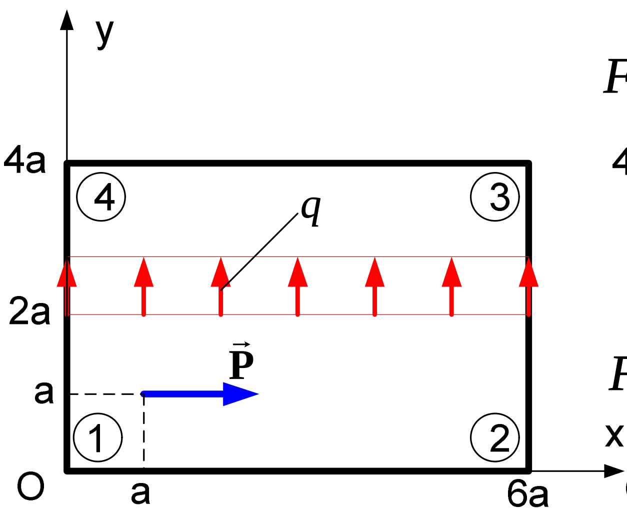
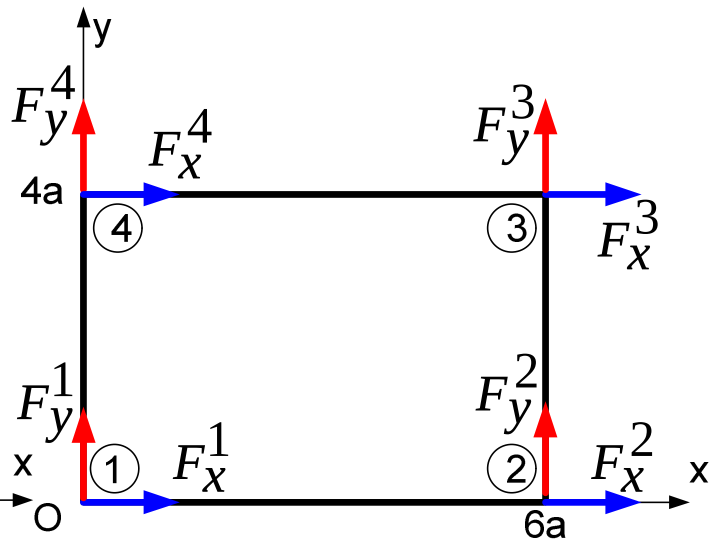
ĐÁP ÁN PHƯƠNG PHÁP PHẦN TỬ HỮU HẠN

Câu 1: 2,5 điểm





Hàm dạng: $N_1 = \dfrac{(6a-x)(4a-y)}{24a^2}; N_2 = \dfrac{x(4a-y)}{24a^2}; N_3 = \dfrac{xy}{24a^2}; N_4 = \dfrac{(6a-x)y}{24a^2};$ **(0,5 đ)**

Tải tương đương quy về các nút là:

$$F_x^1 = N_1(a,a).P = \frac{(6a-a)(4a-a)}{24a^2}P = \frac{5P}{8} = \frac{15}{4}qa; \quad F_y^1 = \int_0^{6a} N_1(x,2a)qdx = \int_0^{6a} \frac{(6a-x)(4a-2a)}{24a^2}qdx = \frac{3}{2}qa$$

$$F_x^2 = N_2(a,a).P = \frac{a(4a-a)}{24a^2}P = \frac{P}{8} = \frac{3}{4}qa; \quad F_y^2 = \int_0^{6a} N_2(x,2a)qdx = \int_0^{6a} \frac{x(4a-2a)}{24a^2}qdx = \frac{3}{2}qa$$

(2 đ)

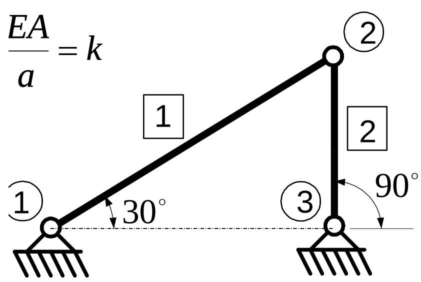
$$F_x^3 = N_3(a,a).P = \frac{a.a}{24a^2}P = \frac{P}{24} = \frac{1}{4}qa; \quad F_y^3 = \int_0^{6a} N_3(x,2a)qdx = \int_0^{6a} \frac{x.(2a)}{24a^2}qdx = \frac{3}{2}qa$$

$$F_x^4 = N_4(a,a).P = \frac{(6a-a)a}{24a^2}P = \frac{5P}{24} = \frac{5}{4}qa; \quad F_y^4 = \int_0^{6a} N_4(x,2a)qdx = \frac{(6a-x)(2a)}{24a^2}qdx = \frac{3}{2}qa$$

Câu 2: 4 điểm

$AB = 2a; BC = a$

$\dfrac{EA}{a} = k$



pht_i	1^i	2^i	k_i	θ_i	(c^2, s^2, cs)
1	1	2	$k_1 = \dfrac{AE}{2a} = \dfrac{k}{2}$	30°	$(c^2 = 3/4; s^2 = 1/4; cs = \sqrt{3}/4)$
2	2	3	$k_2 = \dfrac{AE}{a} = k$	-90°	$(c^2 = 0, s^2 = 1, cs = 0)$

Ma trận độ cứng các phần tử 1 và 2 lần lượt là:

$$\mathbf{K}_1 = \frac{k}{2}\begin{bmatrix} \frac{3}{4} & \frac{\sqrt{3}}{4} & -\frac{3}{4} & -\frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{4} & \frac{1}{4} & -\frac{\sqrt{3}}{4} & -\frac{1}{4} \\ -\frac{3}{4} & -\frac{\sqrt{3}}{4} & \frac{3}{4} & \frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4} & -\frac{1}{4} & \frac{\sqrt{3}}{4} & \frac{1}{4} \end{bmatrix}; \ \mathbf{K}_2 = k\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

Ma trận độ cứng tổng thể là:

$$\mathbf{K}=k\begin{bmatrix} \frac{3}{8} & \frac{\sqrt{3}}{8} & -\frac{3}{8} & -\frac{\sqrt{3}}{8} & 0 & 0 \\ \frac{\sqrt{3}}{8} & \frac{1}{8} & -\frac{\sqrt{3}}{8} & -\frac{1}{8} & 0 & 0 \\ -\frac{3}{8} & -\frac{\sqrt{3}}{8} & \frac{3}{8} & \frac{\sqrt{3}}{8} & 0 & 0 \\ -\frac{\sqrt{3}}{8} & -\frac{1}{8} & \frac{\sqrt{3}}{8} & \frac{9}{8} & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix}$$

Lực nhiệt trong 2 thanh 1 và 2 là:

$$F_{nh}^{1}=\alpha AE\Delta T\begin{Bmatrix} -\cos 30^\circ \\ -\sin 30^\circ \\ \cos 30^\circ \\ \sin 30^\circ \end{Bmatrix}=\alpha AE\Delta T\begin{Bmatrix} -\frac{\sqrt{3}}{2} \\ -\frac{1}{2} \\ \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{Bmatrix};\quad F_{nh}^{2}=\alpha AE\Delta T\begin{Bmatrix} -\cos(-90^\circ) \\ -\sin(-90^\circ) \\ \cos(-90^\circ) \\ \sin(-90^\circ) \end{Bmatrix}=\alpha AE\Delta T\begin{Bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{Bmatrix}$$

Phương trình lực và chuyển vị:

$$\begin{Bmatrix} R_1^x-\frac{\sqrt{3}}{2}\alpha AE\Delta T \\ R_1^y-\frac{1}{2}\alpha AE\Delta T \\ \frac{\sqrt{3}}{2}\alpha AE\Delta T \\ \frac{3}{2}\alpha AE\Delta T \\ R_3^x \\ R_3^y-\alpha AE\Delta T \end{Bmatrix}=k\begin{bmatrix} \frac{3}{8} & \frac{\sqrt{3}}{8} & -\frac{3}{8} & -\frac{\sqrt{3}}{8} & 0 & 0 \\ \frac{\sqrt{3}}{8} & \frac{1}{8} & -\frac{\sqrt{3}}{8} & -\frac{1}{8} & 0 & 0 \\ -\frac{3}{8} & -\frac{\sqrt{3}}{8} & \frac{3}{8} & \frac{\sqrt{3}}{8} & 0 & 0 \\ -\frac{\sqrt{3}}{8} & -\frac{1}{8} & \frac{\sqrt{3}}{8} & \frac{9}{8} & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix}\begin{Bmatrix} u_1=0 \\ v_1=0 \\ u_2 \\ v_2 \\ u_3=0 \\ v_3=0 \end{Bmatrix}$$

Đơn giản hóa

$$\begin{Bmatrix} \frac{\sqrt{3}}{2}\alpha AE\Delta T \\ \frac{3}{2}\alpha AE\Delta T \end{Bmatrix}=k\begin{bmatrix} \frac{3}{8} & \frac{\sqrt{3}}{8} \\ \frac{\sqrt{3}}{8} & \frac{9}{8} \end{bmatrix}\begin{Bmatrix} u_2 \\ v_2 \end{Bmatrix}\Rightarrow\begin{cases} u_2=\sqrt{3}\alpha a\Delta T \\ v_2=\alpha a\Delta T \end{cases}$$ **(2,5 đ)**

Phản lực tại gối nút 1:

$$R_1^x-\frac{\sqrt{3}}{2}\alpha AE\Delta T=k\left(-\frac{3}{8}u_2-\frac{\sqrt{3}}{8}v_2\right)\Rightarrow R_1^x=0$$

$$R_1^y-\frac{1}{2}\alpha AE\Delta T=k\left(-\frac{\sqrt{3}}{8}u_2-\frac{1}{8}v_2\right)\Rightarrow R_1^y=0$$ **(0,5 đ)**

Độ biến dạng trong thanh 1 là:

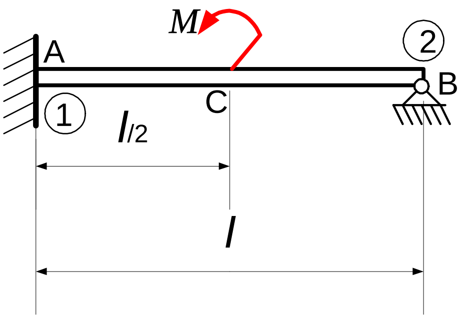
$$\varepsilon=\frac{\sqrt{(l_x^1+u_2-u_1)^2+(l_y^1+v_2-v_1)^2}-l^1}{l^1}=\frac{\sqrt{(2a\cos 30^\circ+u_2-u_1)^2+(2a\sin 30^\circ+v_2-v_1)^2}-2a}{2a}$$

$$\varepsilon=\frac{\sqrt{(2a\cos 30^\circ+\sqrt{3}\alpha a\Delta T-0)^2+(2a\sin 30^\circ+\alpha a\Delta T-0)^2}-2a}{2a}=\alpha\Delta T$$

Ứng suất trong thanh 1 là: $\sigma_1=E\varepsilon=\alpha E\Delta T$ **(1 đ)**

Cách khác tính ứng suất: $\sigma = \dfrac{\sqrt{\left(R_1^x - \frac{\sqrt{3}}{2}\alpha AE\Delta T\right)^2 + \left(R_1^y - \frac{1}{2}\alpha AE\Delta T\right)^2}}{A} = \alpha E\Delta T$

<u>Câu 3:</u> 3,5 điểm



Ma trận độ cứng: $K = \dfrac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$

Tải quy đổi về nút:

$F_1 = -\dfrac{3M}{2L};\quad M_1 = -\dfrac{M}{4};\qquad F_2 = \dfrac{3M}{2L};\quad M_2 = -\dfrac{M}{4}$

$$\begin{Bmatrix} R_y^1 - \dfrac{3M}{2L} \\ M_z^1 - \dfrac{M}{4} \\ R_y^2 + \dfrac{3M}{2L} \\ -\dfrac{M}{4} \end{Bmatrix} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{Bmatrix} w_1 = 0 \\ \theta_1 = 0 \\ w_2 = 0 \\ \theta_2 \end{Bmatrix} \Rightarrow -\frac{M}{4} = \frac{EI}{L^3}.4L^2.\theta_2 \Rightarrow \theta_2 = -\frac{ML}{16EI}$$ **(3 đ)**

Hàm dạng:

$N_1 = \dfrac{1}{L^3}\left(2x^3 - 3Lx^2 + L^3\right);\quad N_2 = \dfrac{1}{L^3}\left(Lx^3 - 2L^2x^2 + L^3x\right); N_3 = \dfrac{1}{L^3}\left(-2x^3 + 3Lx^2\right);\quad N_4 = \dfrac{1}{L^3}\left(Lx^3 - L^2x^2\right)$

Độ võng và góc xoay tại C là:

$$w_C = N_1(x_C)w_1 + N_2(x_C)\theta_1 + N_3(x_C)w_2 + N_4(x_C)\theta_2 = \frac{1}{L^3}\left[L.(\frac{L}{2})^3 - L^2(\frac{L}{2})^2\right]\left(-\frac{ML}{16EI}\right) = \frac{ML^2}{128EI}$$

$$\theta_C = \frac{d}{dx}N_1(x_C)w_1 + \frac{d}{dx}N_2(x_C)\theta_1 + \frac{d}{dx}N_3(x_C)w_2 + \frac{d}{dx}N_4(x_C)\theta_2 = \frac{1}{L^3}\left[3L(\frac{L}{2})^2 - 2L^2(\frac{L}{2})\right]\left(-\frac{ML}{16EI}\right) = \frac{ML}{64EI}$$

(0,5 đ)